Minimum-Snap Trajectory Generator with Error-State LQR

16.31 Project: Final Presentation

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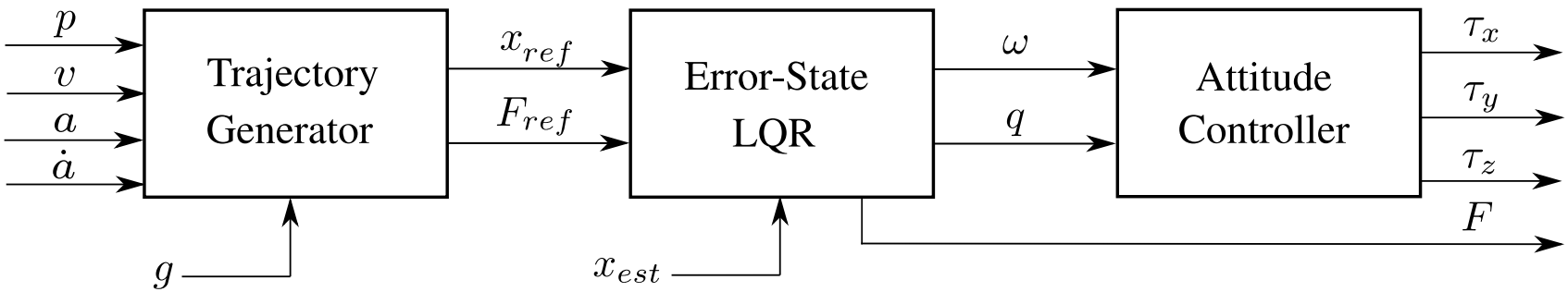
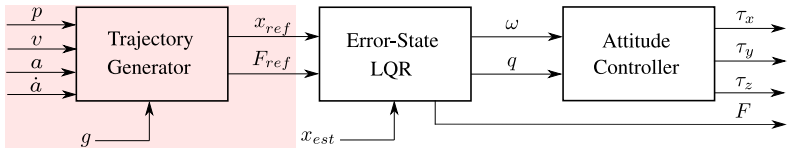
Project Description

Parrot Mambo augmented with a full-state **T**rajectory **G**enerator and **E**rror-state **LQR** controller (**TG-ELQR**):



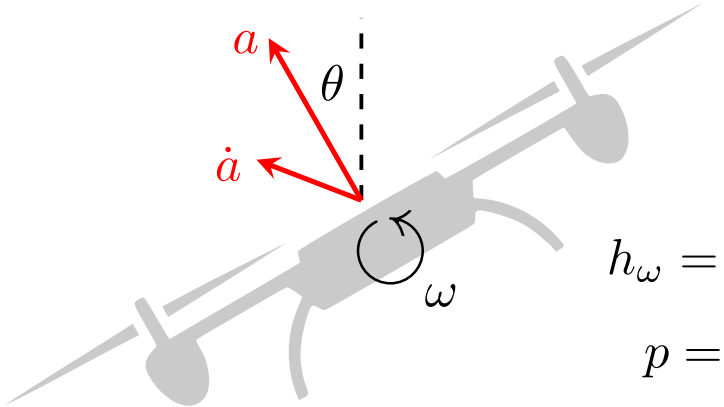
- Attempt more agile flight patterns on Parrot platform
- Full-state trajectory commands with differential flatness [2]
- Commanded attitude + rates with Error-state LQR [3, 1]
- Augmented attitude representation (quaternions) and controller

Minimum-Snap Trajectory Generator



$$\theta = \cos^{-1}(e_3^T \frac{a}{||a||})$$

$$q_I^b = \exp_q(\theta [e_3]_\times \frac{a}{||a||})$$



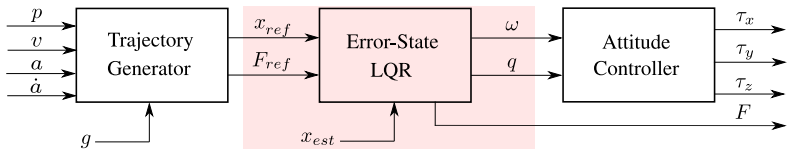
$$h_\omega = \frac{\dot{a} - ((R_I^b)^T e_3 \cdot \dot{a})}{||g - a||} (R_I^b)^T e_3$$

$$p = h_\omega \cdot (R_I^b)^T e_2$$

$$q = -h_\omega \cdot (R_I^b)^T e_1$$

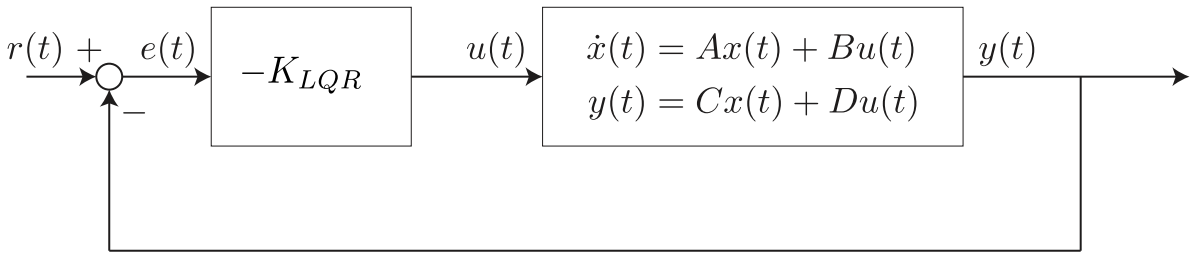
$$r = 0$$

Error-State LQR

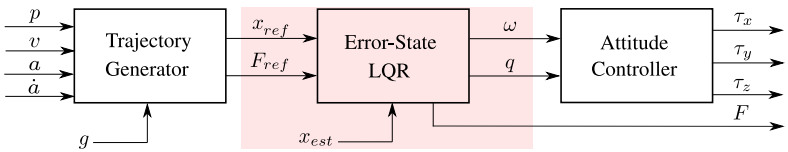


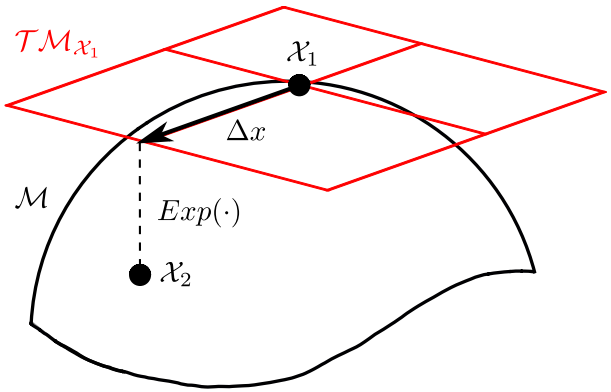
Just like normal LQR, with some quirks:

- The state is acknowledged to exist and evolve on a **manifold**
- The (error) state vector defined as $\tilde{x} \triangleq x \ominus x_c$
- A and B come from Jacobians of the error-state dynamics $f(\tilde{x}, x, u)$ using **Lie derivatives**



Error-State LQR





$$\mathcal{X}_2 = \mathcal{X}_1 Exp(\Delta x) \triangleq \mathcal{X}_1 \oplus \Delta x$$
$$\rightarrow \Delta x = \mathcal{X}_2 \ominus \mathcal{X}_1$$

Figure: Definitions of $\oplus$ and $\ominus$.

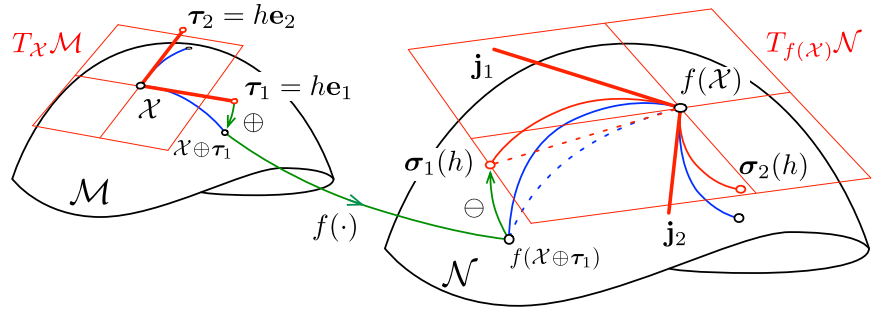
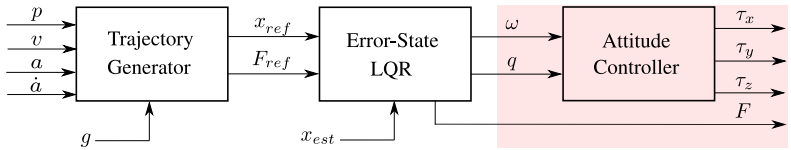


Figure: Definition of Lie Derivative using $\oplus$ and $\ominus$. Figure from [3].

Manifold $\in \mathcal{M}$ + **Tangent Space** $\in \mathcal{TM}$ = **Manifold** $\in \mathcal{M}$

Attitude Controller



Custom Quaternion Attitude Representation Matlab Library

- 3D extension of complex numbers
- Can be interpreted as a kind of axis-angle rotation
- Matlab library handles conversions, special constructor functions, $\oplus$ and $\ominus$ implementations

PID Inner-Loop Attitude Control

- Control off of error in attitude *and angular rates*
- Attitude, angular rate gains roughly same magnitude

Simulation Results

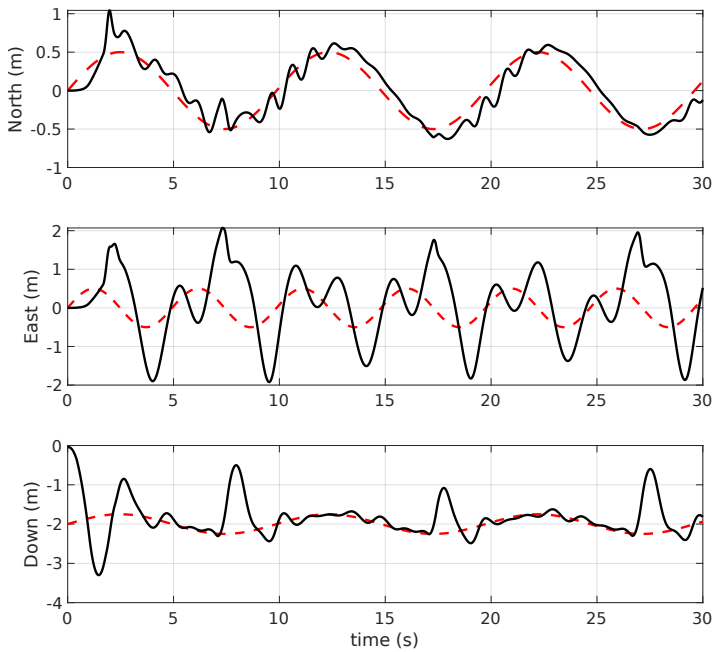


Figure: Position tracking results with default Simulink Parrot Mambo controller.

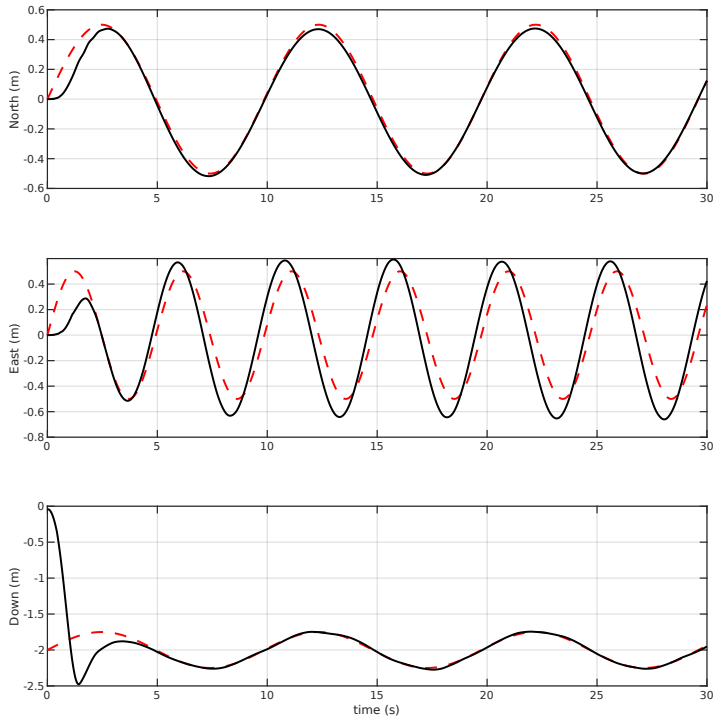


Figure: Postion tracking results with TG-ELQR control.

Simulation Results

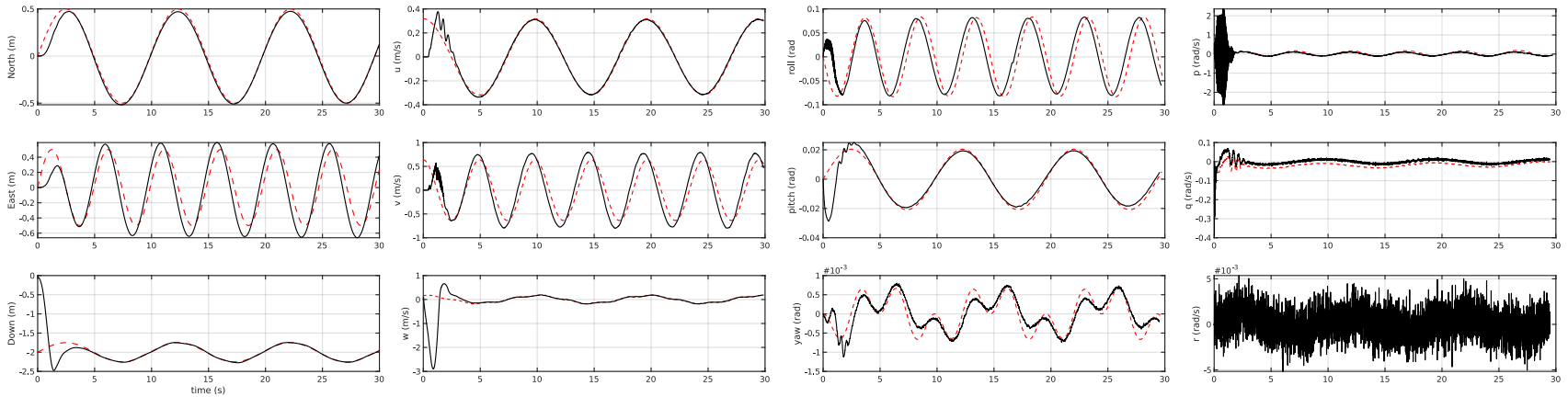
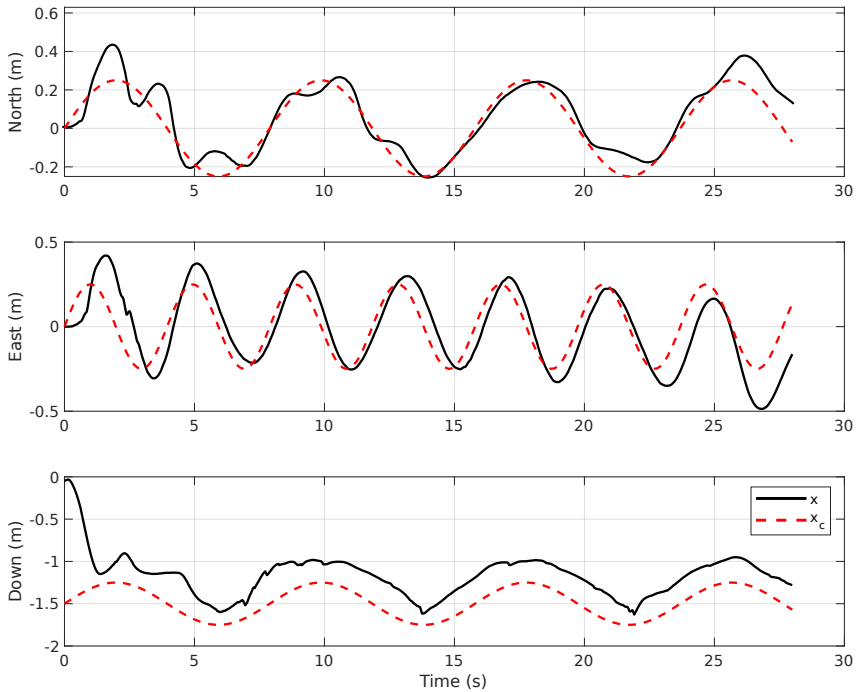


Figure: TG-ELQR tracking performance for *all states* generated by the minimum-snap trajectory generator.

Hardware Validation



VIDEO: Hardware Flight

References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